Oxford Cambridge and RSA

AS Level Mathematics B (MEI)

H630/01 Pure Mathematics and Mechanics

Question Set 5

[2]

1)

Celia states that $n^2 + 2n + 10$ is always odd when n is a prime number.

Prove that Celia's statement is false. [2]

Proof by example

n = prime no.

2 = prime no.

substitute 2 into equation

$2^2 + (2\times2) + 10 = 18$

18 is not odd

$\therefore$ $n^2 + 2n + 18$ is not always odd if n is a prime number.

2)

Fig. 2 shows a quadrilateral ABCD. The lengths AB and BC are 5 cm and 6 cm respectively. The angles ABC, ACD and DAC are 60°, 60° and 75° respectively.

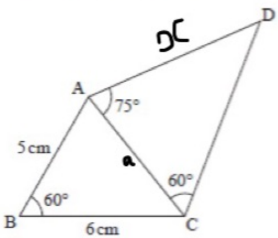


Fig. 2

Calculate the exact value of the length AD. [4]

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 6^2 + 5^2 - 2\times5\times6\cos60$

$a^2 = 61 - 30$

$a^2 = 31$

$a = 5.567$

≈ 5.6

angle $\angle ADC$

$\hookrightarrow 180 - (75 + 60) = 45$

$$\frac{\sin 45}{5.6} = \frac{\sin 60}{AD}$$

$$AD = \sin 60 \div \left(\frac{\sin 45}{5.6}\right)$$

$AD = 6.818$

$AD = 7$ cm 1st

3) Fig. 3 shows a triangle PQR. The vector $\overrightarrow{PQ}$ is $\mathbf{i}+7\mathbf{j}$ and the vector $\overrightarrow{QR}$ is $4\mathbf{i}-12\mathbf{j}$.

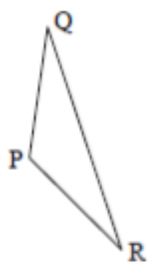


Fig. 3

a) Show that the triangle PQR is isosceles. [3]

For triangle $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RP} = 0$

$\therefore\ i+7j+4i-12j+xi+yj = 0i+0j$

$(1+4+x)i = 0i$

$x+1+4 = 0$

$x = -5$

$(7-12+y)j = 0j$

$7-12+y = 0$

$y = 5$

$\therefore \overrightarrow{RP} = -5i+5j$

$|\overrightarrow{PQ}| = \sqrt{1^2+7^2}$

$= 5\sqrt{2}$

$|\overrightarrow{RP}| = \sqrt{5^2+5^2}$

$= 5\sqrt{2}$

$\therefore$ PQ and RP are equal in length which means they are isocceles

b) Find the position vector of S. ~~[2]~~ −2 marks

exam board error

no "S" in question, $\therefore$ impossible to answer

4) Fig. 4.1 shows part of the curve $y = x^{\frac{1}{2}}$. P is the point (1, 1) and Q is the point on the curve x-coordinate $1 + h$.

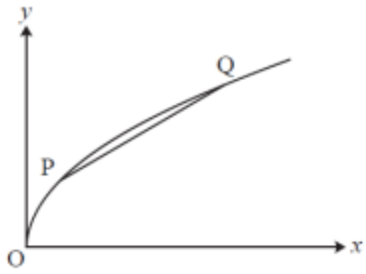


Fig. 4.1

Table 4.2 shows, for different values of h, the coordinates of P, the coordinates of Q, the change in y from P to Q and the gradient of the chord PQ.

x for P	y for P	h	x for Q	y for Q	change in y	gradient PQ
1	1	1	2	1.4142136	0.585786	0.585786
1	1	0.1	1.1	1.048 809	0.048 809	0.488 088
1	1	0.01	1.01	1.004 988	0.004 988	0.498 756
1	1	0.001	1.001	1.000 500	0.000 500	0.499 875

Table 4.2

(a) Fill in the missing values for the case $h = 1$ in the copy of Table 4.2 below. Give your answers correct to 6 decimal places where necessary. **[1]**

b)

Explain how the sequence of values in the last column of Table 4.2 relates to the gradient of the curve $y = x^{\frac{1}{2}}$ at the point P. **[1]**

As h gets smaller the values, move closer to the value of the gradient at P.

c)

Use calculus to find the gradient of the curve at the point P. **[2]**

$$\frac{dy}{dx}(x^{1/2}) = 1/2\, x^{-1/2}$$

$p = (1,1)$

Sustitute $x = 1$

$$1/2\; 1^{-1/2} = 0.5$$

5) **In this question you must show detailed reasoning.**

A curve has equation $y = 4x^3 - 6x^2 - 9x + 4$.

Sketch the gradient function for this curve, clearly indicating the points where the gradient is zero. **[4]**

$\frac{dy}{dx}(4x^3 - 6x^2 - 9x + 4) = 12x^2 - 12x - 9$

$12x^2 - 12x - 9 = 0$

$x = \frac{3}{2}$

$x = -\frac{1}{2}$

turning point

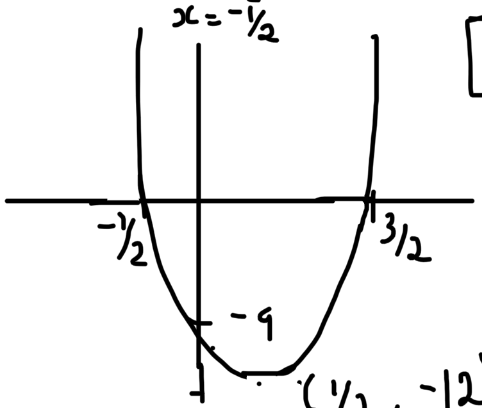
$\frac{dy'}{d'x} = 0$

$\rightarrow 24x - 12 = 0$

$24x = 12$

$x = \frac{1}{2}$

$y = -12$



$\underbrace{(1/2, -12)}_{\text{min point}}$

gradient = 0

b)

Find the set of values of x for which the gradient function is decreasing. Give your answer using set notation. **[2]**

$\frac{dy'}{d'x} < 0$

$24x - 12 < 0$

$24x < 12$

$x < \frac{1}{2}$

$\{x : x < \frac{1}{2}\}$

6) The point A has coordinates $(-1, -2)$ and the point B has coordinates $(7, 4)$. The perpendicular bisector of AB intersects the line $y + 2x = k$ at P.

Determine the coordinates of P in terms of k. [7]

$m = \left(\frac{4 - -2}{7 - -1}\right)$

$m = \frac{3}{4}$

$m' = -\frac{4}{3}$

equation of perpendicular bisector

$y = mx + c$

midpoint of AB $= \left(\frac{7 + -1}{2}\right), \left(\frac{4 + -2}{2}\right)$

$\hookrightarrow (3, 1)$

$1 = \left(-\frac{4}{3} \times 3\right) + c$

$5 = c$

$\therefore\ y = -\frac{4}{3}x + 5$

equation of line $= y = k - 2x$

$-\frac{4}{3}x + 5 = k - 2x$

$\frac{-4x}{3} + 5 + 2x = k$

$\frac{2x}{3} + 5 = k$

$2x + 15 = 3k$

$2x = 3k - 15$

$x = \frac{3k - 15}{2}$

$y = k - 2\left(\frac{3k - 15}{2}\right)$

$y = k - 3k + 15$

$y = 2k + 15$

$\left(\frac{3k - 15}{2}, 2k + 15\right)$

7) **In this question you must show detailed reasoning.**

A student is asked to solve the inequality $x^{\frac{1}{2}} < 4$.

The student argues that $x^{\frac{1}{2}} < 4 \Leftrightarrow x < 16$, so that the solution is $\{x : x < 16\}$.

Comment on the validity of the student's argument. [1]

The students argument is valid for all values of x.

Solve the inequality $\left(\frac{1}{2}\right)^x < 4$. [3]

$$\left(\frac{1}{2}\right)^x < 4$$

$$x > \log_{1/2} 4$$

$$x > -2$$

c)

Show that the equation $2\log_2(x+8) - \log_2(x+6) = 3$ has only one root. [5]

$$2\log_2(x+8) = \log_2(x+8)^2$$

$$\log_2(x+8)^2 - \log_2(x+6) = 3$$

$$\log_2\left(\frac{(x+8)^2}{x+6}\right) = 3$$

$$\frac{(x+8)^2}{(x+6)} = 2^3$$

$$(x+8)^2 = 8(x+6)$$

$$x^2+16x+64 = 8x+48$$

$$x^2+8x+16=0$$

discriminant

$$b^2-4ac$$

$$8^2 - 4\times1\times16$$

$$64-64=0$$

because the discriminant is zero there is only one root.

8) **In this question you must show detailed reasoning.**

Fig. 8 shows part of the graph of $y = x^2 + \frac{1}{x^2}$.

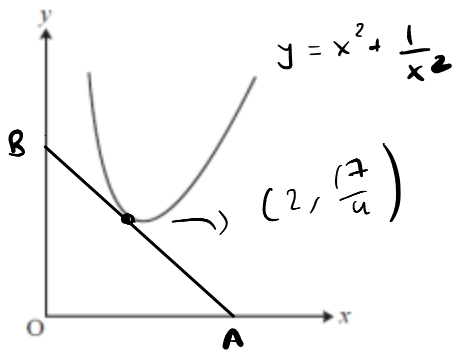


Fig. 8

The tangent to the curve $y = x^2 + \frac{1}{x^2}$ at the point $(2, \frac{17}{4})$ meets the x-axis at A and meets the y-axis at B. O is the origin.

Find the exact area of the triangle OAB. **[6]**

Tangent to the curve at $(2, 17/4)$

$\frac{dy}{dx}$ of $x^2 + x^{-2} = 2x - 2x^{-3}$

at $x = 2$: $2(2) - 2(2)^{-3} = 4 - \frac{1}{4} = \frac{15}{4}$

Equation of tangent:

$$y = \frac{15}{4}x + c$$

$$\frac{17}{4} = \frac{15}{4}(2) + c \implies \frac{17}{4} = \frac{15}{2} + c$$

$$-\frac{13}{4} = c$$

$$\therefore y = \frac{15}{4}x - \frac{13}{4}$$

\-

finding A

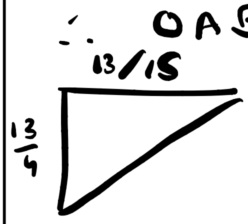
$y=0$ $\qquad \frac{13}{4} = \frac{15}{4}x \qquad \therefore A = \left(13/15, 0\right)$

$\frac{13}{15} = x$

Finding B

$x=0 \qquad y = -\frac{13}{4} \qquad B = \left(0, -\frac{13}{4}\right)$

$\therefore$ OAB



$\frac{1}{2}\left(13/15\right)\left(\frac{13}{4}\right)$

b) Use calculus to prove that the complete curve has two minimum points and no maximum point. [6]

Stationary points

$2x - 2x^{-3} = 0$

$2x = \frac{2}{x^3}$

$2x^4 = 2$

$x^4 = 1$

$x^4 - 1 = 0$

$x = 1$

OR $x = -1$

$\frac{d^2y}{dx^2} = 2 + 6x^{-4}$

for $x = 1$

$2 + 6(1)^{-4} = 8$

$8 > 0 \quad \therefore$ minimum point

for $x = -1$

$2 + 6(-1)^{-4} = 8$

$8 > 0 \quad \therefore$ minimum point

$\therefore$ 2 minimum points and no maximum points

$-\frac{61}{126}$ 9

$= 1.41 \text{ (3sf)}$

Total Marks for Question Set 5: 49 marks - 2marks → only 47 marks